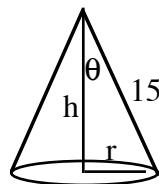


Application Problem - The Cone Problem

Shown at the right is a cone with a slant height of 15 cm. Let's explore the relationship between the **volume** of the cone ($V = \frac{1}{3}\pi r^2 h$) and the **angle** θ .



What would the cone look like if angle θ was a very **small** angle? (Remember, the slant height does not change!) Draw it to the right. Do you think the volume would be small or large? _____

What would the cone look like if angle θ was a very **large** angle? Draw it to the right. Do you think the volume would be small or large? _____

Let's take a small angle for θ , for example **5 degrees**. Since the volume of the cone is dependent on the radius and the height of the cone, we need to calculate the radius and the height if the angle is 5 degrees.

Write a trig equation you could use to determine the **radius** of the cone. _____
Solve the equation to find the radius.

If $\theta = 5$ degrees, $r =$ _____.

Write a trig equation you could use to find the **height** of the cone. _____
Solve the equation to find the height.

If $\theta = 5$ degrees, $h =$ _____

Now, let's see what would happen if the angle θ gets bigger. Evaluate the **radius**, the **height**, and the **volume** of the cone for the different values of the angle θ listed in the chart below. Watch what happens to the **volume**!

Angle	Radius	Height	Volume
5°			
20°			
35°			
50°			
65°			
80°			

From your chart, where does it appear that the **maximum** value of the volume of the cone occurs?

The **maximum volume** of the cone is _____ and it occurs when the **angle** θ is _____.

Create a **scatterplot** to graphically show the relationship between the **angle** θ and the **volume** of the cone. Using the values from your chart on the preceding page, put the values of θ in L1, and the values of the volume in L2.

To set up the appropriate window to see the data, we need to choose reasonable values for **Xmin**, **Xmax**, **Ymin**, and **Ymax**. First, what do the horizontal values (the independent variable) represent?

What values of θ make sense in the problem? _____

Let: **Xmin**=_____ **Xmax**=_____

What do the vertical values (the dependent variable) represent in the problem?

Let: **Ymin**=_____ **Ymax**=_____

To determine the **exact** value of θ that maximizes the volume, we need to write a function for the volume **V** in terms of the angle θ . Use trig to write the **radius** of the cone in terms of θ .

r= _____

Use trig to write the **height** of the cone in terms of θ .

h = _____

Using these two equations write an equation for the volume of the cone in terms of θ .

V(θ) = _____

Put this equation into **Y1** of your calculator (with θ replaced by **x**) and graph the volume function.

Of course, the function should go through the data points of your scatterplot. (If it does not, determine why not!)

Now use your grapher to complete the following:

1. The **maximum volume** for a cone with slant height of 15 cm is _____ and it occurs when the angle is _____.
2. If the volume of the cone is 300 cm^3 , what would the angle θ be? _____
3. If the slant height was 60 cm instead of 15 cm, how would this affect the **volume** of the cone? How would it affect the **angle** θ ? Let the slant height be 60 cm and the angle θ be x degrees.

The **radius** in terms of x would be $r =$ _____, the **height** would be $h =$ _____ and the **volume** would be:

$$V(\theta) = \text{_____}.$$

Put this equation into **Y1** of your calculator, **set an appropriate window**, and graph the volume function.

The **maximum volume** for a cone with slant height of 60 cm is _____ and it occurs when the angle is _____. What is the relationship between the maximum volume when the slant height is 60 cm and the maximum volume when the slant height is 15 cm?

(Don't just say it is larger! You are to find a relationship between the two volumes.)

How do the angles compare? _____